1. Evaluate $\cos 20^\circ - \cos 40^\circ + \cos 60^\circ - \cos 80^\circ$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) $\frac{1}{4}$

2. If x and y are real numbers such that $x^2 > y^2$, which of the following inequalities must hold?

(A) $x > y$ (B) $x + y > 0$ (C) $x^2 > y$ (D) $y^2 > x$ (E) $|x| > |y|$

3. Let n be a 2017-digit number such that every 2-digit number formed by two consecutive digits of n is divisible either by 17 or by 23. If the last digit of n is 1 then what is the first digit of n?

(A) 2 (B) 3 (C) 4 (D) 6 (E) 9

4. Given any triangle, in a single move you are allowed to change the length of one of its sides so that the result is still a triangle. What is the minimal number of such moves required to transform an equilateral triangle with side 100 to an equilateral triangle with side 1?

(A) 19 (B) 14 (C) 11 (D) 12 (E) 10

5. Find the number of integers between 1 and 100, inclusive, that can be written as a sum of non-negative integers such that digits 0-9 are all used exactly once. For example, 90 = 0 + 1 + 52 + 3 + 4 + 6 + 7 + 8 + 9.

(A) 6 (B) 7 (C) 9 (D) 11 (E) 13

6. Suppose $f(n+1,m) = f(n,m) + m + 1$, $f(n,0) = n$, $f(n,m) = f(m,n)$ for all n,m.

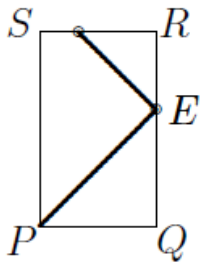
What is $f(2017,5)$?

(A) 12,110 (B) 12,105 (C) 12,100 (D) 12,103 (E) 12,107

7. How many negative roots does the equation $x^4 - 5x^3 - 4x^2 - 2017x + 4 = 0$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. On a rectangular table **PQRS** of **5** units long and **3** units wide, a ball is rolled from point **P** at an angle of **45** toward the point **E**, and bounces off **SR** at an angle of **45** as shown below. The ball continues to bounce off the sides at 45 until it reaches **R**. How many times has the ball bounced?



(A) 4 (B) 5 (C) 6 (D) 7 (E) 9

9. Let $S(n)$ be the sum of the digits of an integer n and let N denote the smallest positive integer such that $N + S(N) + S(S(N)) = 99$. What is $S(N)$?

(A) 9 (B) 10 (C) 12 (D) 15 (E) 18

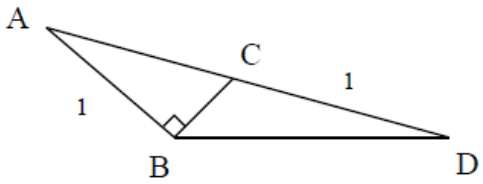
10. Suppose that x and y are two real numbers which satisfy the equation

$$\left(x+\sqrt{x^2+1}\right)\left(y+\sqrt{y^2+1}\right)=1$$

What is the most precise conclusion that one can make?

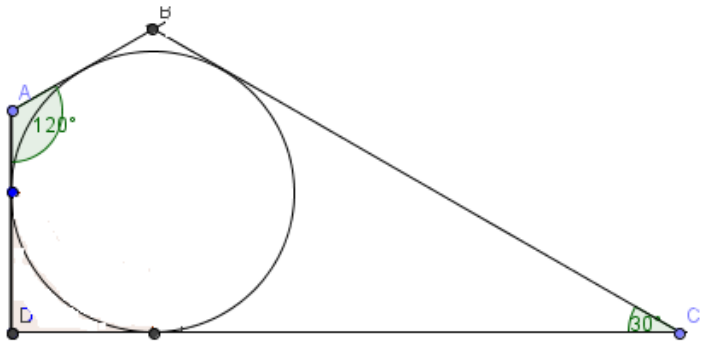
(A) $x = y = 0$ (B) $x + y = 0$ (C) $x + y \le 0$ (D) $x - y = 0$ (E) $xy = 0$

11. Assume that $|AB| = |CD| = 1$, $\measuredangle ABC = 90$, and $\measuredangle CBD = 30$ in the figure. Find $|AC| = ?$.



(A) $\sqrt[3]{2}$ (B) $\sqrt{2}$ (C) $\frac{1+\sqrt{2}}{2}$ (D) $\frac{1+\sqrt{3}}{2}$ (E) $\frac{5}{4}$

12. Let $ABCD$ be an circumscribed quadrilateral such $\measuredangle A = \measuredangle B = 120^0$, $\measuredangle C = 30^0$, and $|BC| = 2$, what is $|AD|$?



(A) $\sqrt{2}-3$ (B) $\sqrt{3}-1$ (C) $\sqrt{6}-\sqrt{2}$ (D) $2-\sqrt{2}$ (E) $3-\sqrt{3}$

13. Let A be a subset of $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$ satisfying the following property: whenever you choose a subset B consisting of 3 elements of A, there are at least two elements in B that have a common factor larger than 1. What is the largest possible number of elements in A?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

14. Let a, b and c be positive integers such that $abc + ab + ac + bc + a + b + c = 1000$.

Find $a + b + c = ?$

(A) 43 (B) 42 (C) 31 (D) 28 (E) 24

15. Find the radius of the circle inscribed in a triangle whose sides are 8, 15, and 17.

(A) 2.4 (B) 2.6 (C) 2.8 (D) 2.9 (E) 3.0

16. How many integers n with $1 \le n \le 100$ can be written in the form $n = [x] + [2x] + [3x]$, where x is a real number and $[x]$ denotes the largest integer less than or equal to x?

(A) 17 (B) 33 (C) 50 (D) 67 (E) 83

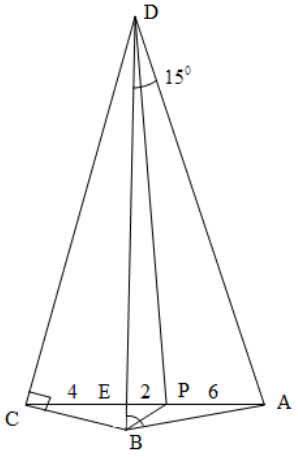
17. Suppose f is a function satisfying $f(x + f(x)) = 4f(x)$ and $f(1) = 4$. Find $f(21) = ?$

(A) 16 (B) 21 (C) 64 (D) 105 (E) Impossible to find it

18. Suppose that $M = 1^{-6} + 2^{-6} + 3^{-6} + 4^{-6} + 5^{-6} + \cdots$ and $N = 1^{-6} + 3^{-6} + 5^{-6} + 7^{-6} + \cdots$. Find $\frac{M}{N} = ?$

(A) $\frac{63}{64}$ (B) $\frac{64}{63}$ (C) $\frac{128}{127}$ (D) $\frac{127}{128}$ (E) 1

19. Let *ABCD* be a convex quadrilateral such that $<ADB=15^0$, $<BCD=90^0$. The diagonals of quadrilateral are perpendicular at E. Let *P* be a point on $|AE|$ such that $|EC| = 4$, $|EA| = 8$ and $|EP| = 2$. What is $<PBD$



(A) 15^0 (B) 30^0 (C) 45^0 (D) 60^0 (E) 75^0

20. Let $S = \left\{\frac{1}{256}, \frac{1}{32}, \frac{1}{4}, 2, 16, 128, 1024\right\}$. How many real numbers can be written as a product of three distinct elements of S?

(A) 13 (B) 15 (C) 20 (D) 21 (E) 35

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