الجامعة الإسلامية العالمية ماليزيا
INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA

(Company No. 101067-P)

Garden of Knowledge and Virtue

IIUM Mathematics Competition (IMC 2019)

FINAL ROUND: SUBJECTIVE SOLUTIONS

18th SEPTEMBER 2019

2 ½ HOURS (09:30 am – 12:00 pm)

INSTRUCTIONS TO STUDENTS:

1. This paper consists of 6 printed pages (including cover page) with **5 subjective questions**.
2. Answer **ALL** questions in this booklet.
3. Students are allowed to use pencil, pen, eraser, and ruler **ONLY**.
4. Students are **NOT** allowed to bring a book, calculator, briefcase, hand phone, protractor, compass, etc.
5. Students are **NOT** allowed to discuss the questions during the examination.

Name:____________________ School:__________________

1. Five students have the first names Clark, Donald, Jack, Robin and Steve, and have the last names (in a different order) Clarkson, Donaldson, Jackson, Robinson and Stevenson. It is known that Clark is 1 year older than Clarkson, Donald is 2 years older than Donaldson, Jack is 3 years older than Jackson, Robin is 4 years older than Robinson.

Who is older, Steve or Stevenson and what is the difference in their ages?

(12 Marks)

Solution. The sum of ages of Clark, Donald, Jack, Robin and Steve is equal to the sum of ages of Clarkson, Donaldson, Jackson, Robinson and Stevenson. Hence Stevenson is older than Steve, and the difference is 1 + 2 + 3 + 4 = 10 years.

$c + 1 = C,$

$d + 2 = D,$

$j + 3 = J,$

$r + 4 = R$

$s + x = S$

$c+d+j+r+s+1+2+3+4+x = C+D+J+R+S$

$x = -10$

S (Steve) is younger than s (Stevenson) by 10 years

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2 Let C(n) be the number of prime divisors of a positive integer n. (For example, C(10) = 2, C(11) = 1, C(12) = 2).

Consider set S of all pairs of positive integers (a, b) such that $a \neq b$ and $C(a + b) = C(a) + C(b)$.

Is set S finite or infinite? . **(12 Marks)**

Solution. The set of pairs is infinite.

Example 1 . $a = 2^k$, $b = 2^{k+1}$, $(a + b) = 3 \cdot 2^k$, $k = 1, 2, \ldots$. Then C(a) = 1, C(b) = 1, C(a + b) = 2.

Example 2 (based on different idea). Let a = p, b = 5p, (a+b) = 6p = $2 \cdot 3 \cdot p$. Let $p \neq 2, 3, 5$ is a prime. Then, C(a) = 1, C(b) = 2, C(a + b) = 3.

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3. A circle touches sides AB, BC, CD of a parallelogram ABCD at points K, L, M respectively. Prove that the line KL bisects the height of the parallelogram drawn from the vertex C to AB. **(12 Marks)**

Solution. Let CH be the height dropped from the vertex C to the side AB. Let P and Q be the points of intersection of line KL with CD and CH respectively.

Observe that BK = BL and CL = CM (as tangents to a circle) and that triangles BLK and PCL are similar (angle–angle criteria). Therefore the triangle LPC is isosceles, CP = CL and therefore PC = CM. Hence, CH is the midline in triangle PMK, so CQ = MK/2 =CH/2.

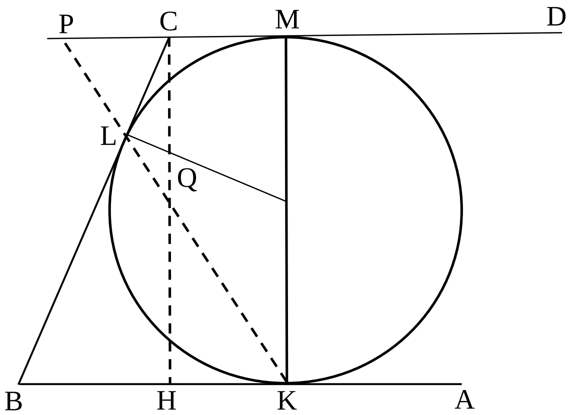

Fig.1

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4. Find the least positive integer for which the product $1260 \cdot n$ is the cube of a natural number.
(12 Marks)

Solution. 1260=2×2×3×3×5×7. Then n=2×3×5×5×7×7=7350.

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5. Solve the following radical equation

$$\sqrt{3x+13}=x+1$$

(12 Marks)

Solution Square the radical equation we have
$3x+13=x^2+2x+1$, and $x^2 - x - 12=0$; $x=4$ or $x=-3$.

Second root $x=-3$ is extraneous, since $-3+1<0$. Answer $x=4$.