Name:____________________ School:__________________

1. Find non-negative integer x, y, z which satisfy the following equation

i) $1009^x + y^2 z = 2018$ **(10 Marks)**

ii) $1009^x + yz = 2018$ **(10 Marks)**

Solution. *i*) It is evident that $x=0$ or $x=1$.

If $x=0$ then $y^2 z = 2017$

Since 2017 is prime number then $y=1$ and $z=2017$.

If $x=1$ then $y^2 z = 1009$.

Since 1009 is prime number then $y=1$ and $z=1009$.

ii) It is evident that $x=0$ or $x=1$.

If $x=0$ then $yz = 2017$

Since 2017 is prime number then $y=1$ and $z=2017$ or $y=2017$ and $z=1$.

If $x=1$ then $yz = 1009$.

Since 1009 is prime number then $y=1$ and $z=1009$ or $y= 1009$ and $z=1$.

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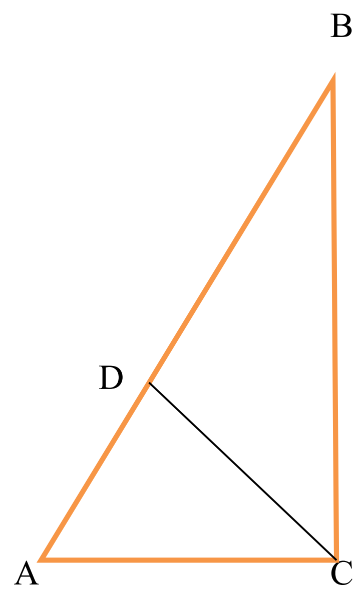
2 For right - angle triangle ABC with $\angle C = 90°$, $\angle B = 30°$ and c=10, find all the altitudes, medians and inner bisectors. **(20 Marks)**

Solution: Firstly find b=5 and a=5$\sqrt{3}$.



It is evident $h_b = a = 5\sqrt{3}$ and $h_a = b = 5$.

Then $h_c = \dfrac{ab}{c} = \dfrac{25\sqrt{3}}{10} = \dfrac{5\sqrt{3}}{2}$.

Now $m_b = \sqrt{a^2 + \left(\dfrac{b}{2}\right)^2} = \sqrt{75 + \dfrac{25}{4}} = \dfrac{5\sqrt{13}}{2}$

$m_a = \sqrt{b^2 + \left(\dfrac{a}{2}\right)^2} = \sqrt{25 + \dfrac{75}{4}} = \dfrac{5\sqrt{7}}{2}$. $m_c = \dfrac{c}{2} = 5$.

$l_a = \dfrac{5}{Cos\ 30°} = \dfrac{10\sqrt{3}}{3}$.

$l_b = \dfrac{5\sqrt{3}}{Cos\ 15°} = \dfrac{5\sqrt{3}}{\sqrt{\dfrac{1 + Cos 30°}{2}}} = \dfrac{10\sqrt{3}}{\sqrt{2+\sqrt{3}}} = 10\sqrt{6 - 3\sqrt{3}}$.

Using the Sine rule for triangle Δ ACD we have

$$l_c = \frac{5 \cdot Sin60^\circ}{Sin75^\circ} = \frac{5\sqrt{3}}{2(Sin45^\circ \cdot Cos30^\circ + Sin30^\circ \cdot Cos45^\circ)} = \frac{5\sqrt{6}(\sqrt{3}-1)}{2}.$$

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3. Find the domain of the following functions

i) $f(x) = \sqrt{x-1} + \sqrt{6-x} + \dfrac{2}{x^2 - 5x + 6}$ **(10 Marks)**

ii) $g(x) = \sqrt{x-1} + \sqrt{6-x} + \dfrac{2}{x^2 - 5x + 6} + \sqrt{Sin\,\pi x}$ **(10 Marks)**

Solution: *i*) We have the following inequalities

$$x \geq 1, \quad x \leq 6 \quad \text{and} \quad x \neq 2,3.$$

Solving them we have that domain of $f(x)$ is $[1,2) \cup (2,3) \cup (3,6]$.

ii) We have the following inequalities

$$x \geq 1, \quad x \leq 6\,, x \neq 2,3 \quad \text{and} \quad x \in [0,1] \cup [2,3] \cup [4,5] \cup [6,7]$$

Solving them we have that domain of $g(x)$ is $\{1\} \cup (2,3) \cup [4,5] \cup \{6\}$.