

Name: \_\_\_\_\_

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1. Determine all pairs of non-negative integers  $(m, n)$  which are solutions to the equation
- $$m^3 + n^3 = m^2n + mn^2 + 2013.$$

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2. Let two functions  $f(x)$  and  $g(x)$  satisfy the following conditions

$$f(2x+1) + g(3-x) = x$$

and

$$f\left(\frac{3x+5}{x+1}\right) + 2g\left(\frac{2x+1}{x+1}\right) = \frac{x}{x+1}$$

for all real numbers  $x \neq 1$ . What is  $f(2013)$ ?

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3. Let  $\triangle ABC$  be a triangle with  $|AB|=4$ ,  $|AC|=3$  and  $\angle BAC = 90^\circ$ . Let  $E$  and  $D$  be points on sides  $[AB]$  and  $[BC]$  respectively, such that  $\angle EDC = 90^\circ$ . If the area of the triangle  $\triangle BDE$  is equal to area of quadrangle  $EDCA$ , then what is  $|DC|$ ?

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4. Solve the equation

$$\sqrt[3]{6+x} + \sqrt[3]{6-x} = \sqrt[3]{3}$$

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5. There is a positive integer  $A$ . Two operations are allowed: increasing this number by 3 and deleting a digit equal to 1 from any position.

- (i) Is it possible to obtain 212 by applying these operations several times to  $A=211$  ?
- (ii) Is it possible to obtain 2013 by applying these operations several times to  $A=2012$  ?
- (iii) Is it always possible to obtain  $A + 1$  by applying these operations several times to any  $A$ ?