

الجامعة الإسلامية العالمية ماليزيا

INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA

IIUM Mathematics Competition (IMC 2012)

Preliminary Stage

MULTIPLE CHOICE QUESTIONS

**This Question Paper Consists of 7 Printed Pages
including cover page with 20 Questions**

**Department of Computational and Theoretical Sciences
Kulliyyah of Science**

1. If geometric mean of numbers a and b is equals to 3, and their arithmetic mean is equals to 6, find arithmetic mean of a^2 and b^2 . Here arithmetic mean of a and b is $\frac{a+b}{2}$ and geometric mean is \sqrt{ab} .

- (A) 67 (B) 65 (C) 63 (D) 61 (E) 57

2. If real numbers x and y satisfy to following two equalities

$$x^3 - 3x^2y = 3$$

$$y^3 - 3xy^2 = 11$$

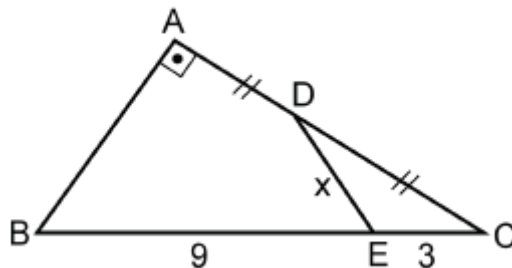
find $x - y$.

- (A) 3 (B) 2 (C) 1 (D) - 2 (E) - 3

3. Let for two-digit positive numbers a and b we have $\frac{a!}{b!} = 132$. Here $a!$ (a factorial) is the product of all positive integers less than or equal to a . Choose correct answer for $a + b$.

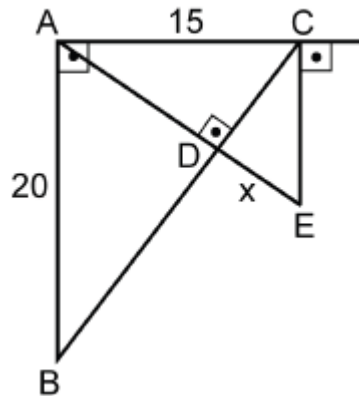
- (A) 22 (B) 23 (C) 24 (D) 25 (E) 26

4. Let ABC be a triangle with $BA \perp AC$. If $AD = DC$, $EC = 3$, and $BE = 9$, find $DE = x$.



- (A) $\frac{7}{3}$ (B) $\frac{10}{3}$ (C) 2 (D) $\frac{3}{3}$ (E) 4

5. If $AB \perp AC$, $CE \perp CA$, $AE \perp BC$, $AB = 20$, $AC = 15$, find $DE = x$.



- (A) $\frac{15}{2}$ (B) $\frac{25}{3}$ (C) $\frac{32}{3}$ (D) $\frac{36}{5}$ (E) $\frac{27}{4}$

6. For given different prime numbers p and q let $a = p^4 \cdot q^2$ and $b = p^2 \cdot q^3$

Find the greatest common divisor of the numbers a and b .

- (A) $p^5 \cdot q^4$ (B) $p^4 \cdot q^3$ (C) $p^3 \cdot q^4$ (D) $p^2 \cdot q^2$ (E) $p^2 \cdot q^3$

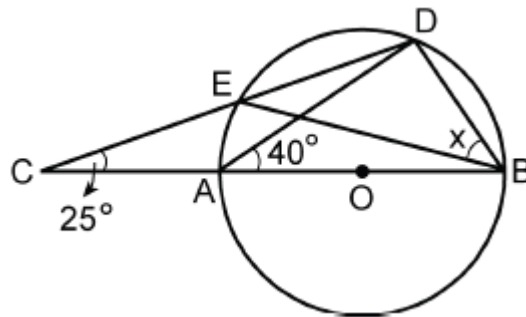
7. Two perpendicular chords of a circle are at distances 2012 and 1 respectively from the center. These two chords divide the circle into four pieces. Consider the sum of areas of the largest and the smallest pieces, and the sum of areas of the other two pieces. Find the difference between these two sums.

- (A) 4024 (B) 8048 (C) 2012 (D) 2013 (E) 2010

8. In how many ways one can represent the number 2012 as the sum of one or more consecutive integers?

- (A) 4 (B) 5 (C) 3 (D) 1 (E) 2

9. If AB is a diameter and $\angle DCB=25^\circ$, $\angle DAB=40^\circ$, find $\angle EBD=x$.

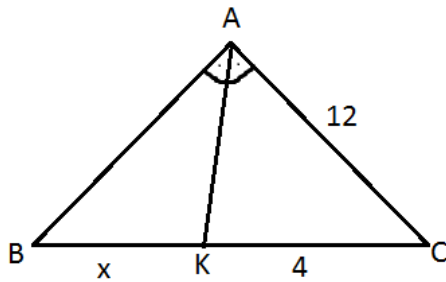


- (A) 25 (B) 30 (C) 35 (D) 40 (E) 45
)

10. Let x be a real number with $|x| \leq 4$. Find the sum of all integer y which satisfy to following equality $2x + 3y = 1$.

- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

11. Let ABC be a triangle with bisector $[AK]$,



where $|AC| = 12$ cm ; $|KC| = 4$ cm and $|BK| = x$.

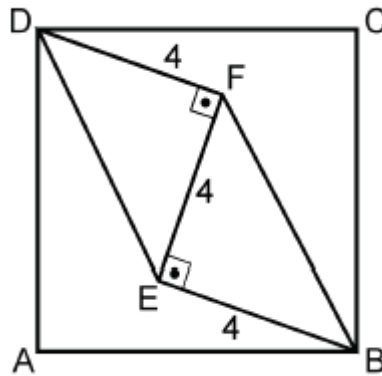
If the perimeter of the triangle ABC is equal to 44cm, find x ?

- (A) 6 (B) 7 (C) 8 (D) 11/2 (E) 13/2

12. Let on the set of positive integer numbers the operations “*” and “ Δ ” are defined as follows:
 $x*y = \text{GCD}(x,y)$ and $x\Delta y = \text{LCM}(x,y)$, where GCD denotes greatest common divisor and LCM denotes least common multiple. Find $18*(12\Delta 4)$.

- (A) 2 (B) 3 (C) 6 (D) 8 (E) 9

13. Let ABCD be a square. If $DF \perp FE$, $FE \perp EB$, and $DF = FE = EB = 4$, find the area of ABCD.



- A) 50 (B) 40 (C) 48 (D) 32 (E) 36

14. Let a function f for integers $n \geq 1$ is defined by equality

$$f(n) = 2 \cdot f(n - 1) + 1$$

If $f(0) = 1$, find $f(2)$.

- (A) 8 (B) 7 (C) 6 (D) 5 (E) 4

15. For natural numbers x and y we have following

$$10 \overline{\begin{matrix} m \\ x \end{matrix}} \qquad 15 \overline{\begin{matrix} n \\ y \end{matrix}}$$

$$\underline{\qquad\qquad\qquad} \qquad \underline{\qquad\qquad\qquad}$$

$$2 \qquad\qquad\qquad 3$$

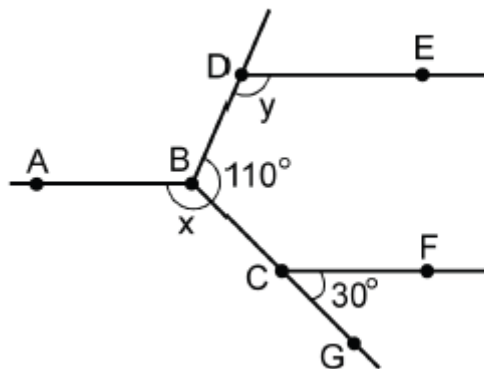
What is the remainder when their product $x \cdot y$ is divided by 5 ?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. If $f(x) = x^2$ and $g(x) = 2x - 1$, find $g(f(2))$

- (A) 0 (B) 3 (C) 5 (D) 7 (E) 9

17 If $DE \parallel AB \parallel CE$, $\angle DCB = 110^\circ$, $\angle FCG = 30^\circ$, $\angle ABC = x$, and $\angle EDB = y$, find $x - y$.



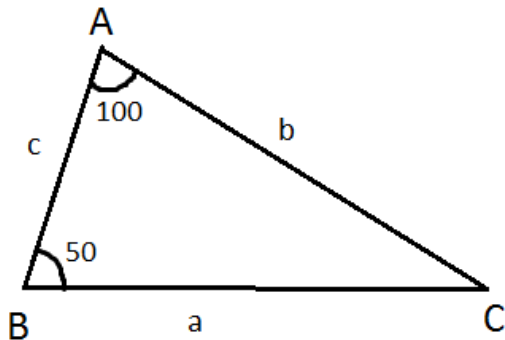
- (A) 50 (B) 45 (C) 40 (D) 35 (E) 30

18. The sum of three- digit number ABC and two- digit number AB is equal to 392.

Find $A + B + C$.

- (A) 7 (B) 9 (C) 11 (D) 15 (E) 19

19. Let ABC be a triangle with $m(\angle ABC) = 50^\circ$ and $m(\angle CAB) = 100^\circ$.



According to this information what is $\frac{|a-b|+|b-c|+|c-a|}{2}$?

- (A) $a - c$ (B) $a - b$ (C) $b - c$ (D) $b - a$ (E) $c - b$

20. The farmer has six containers with volumes 5, 9, 12, 15, 23 and 45 liters respectively. He filled out them with sunflower and olive oils. If amount of sunflower oil is four times more than amount of olive oil, which of these containers left empty?

- (A) 5 (B) 9 (C) 12 (D) 15 (E) 23

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