

1. Let a, b, c be real numbers such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2 \quad \text{and} \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1.$$

Find the value of $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}$.

- (A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) -3 (D) 3 (E) 1
2. Find the number of diagonals that can be drawn in a convex polygon of n ($n \geq 4$) sides.
- (A) $\frac{n(n-1)}{2}$ (B) $\frac{n(n-2)}{2}$ (C) $\frac{n(n-3)}{2}$ (D) $\frac{n(n+1)}{2}$ (E) $\frac{n(n+2)}{2}$
3. In a triangle $\triangle ABC$, $AB = 41$, $AC = 9$, $BC = 40$. Find the radius of the inscribed circle of ABC .
- (A) 2 (B) 3 (C) 4 (D) 4.5 (E) 5
4. How many integers from 1 to 2007 have the sum of their digits divisible by 5?
- (A) 399 (B) 400 (C) 401 (D) 402 (E) 403
5. 20 football teams take part in a tournament. M matches have been played and it is found that
- between any two teams at most one match has been played and,
 - among any three teams at least one match has been played between two of them
- What is the smallest possible value of M ?
- (A) 20 (B) 40 (C) 60 (D) 80 (E) 90
6. Points D and E are points inside an equilateral triangle ABC such that $DE = 1$, $AD = EA = \sqrt{7}$,
 $BD = EC = 2$.
Find the length of AB .
- (A) $\frac{5 + \sqrt{13}}{2}$ (B) $\frac{5 + \sqrt{14}}{2}$ (C) $\frac{5 + \sqrt{15}}{2}$ (D) 4.5 (E) $\frac{5 + \sqrt{17}}{2}$
7. A book has 30 chapters. The length of the each chapter are 1, 2, ..., 30 pages respectively. Chapter one starts from page 1 of the book and each chapter starts from a new page.
At most how many chapters can start from an odd-numbered page?
- (A) 22 (B) 23 (C) 24 (D) 15 (E) 1

8. The length of the three medians AD, BE and CF of a triangle $\triangle ABC$ are 9, 12 and 15 respectively. Find the area of $\triangle ABC$

(A) 68 (B) 70 (C) 72 (D) 74 (E) 90

9. An $m \times n \times p$ rectangular box has half the volume of an $(m+2) \times (n+2) \times (p+2)$ rectangular box, where m, n, p are integers and $m \leq n \leq p$.

What is the largest possible value of p ?

(A) 110 (B) 120 (C) 130 (D) 140 (E) 150

10. Determine the number of acute-angled triangles with consecutive integer sides and of perimeter not exceeding 100.

(A) 26 (B) 27 (C) 28 (D) 29 (E) 30

11. What is the largest positive integer n for which there is a unique integer k such that

$$\frac{8}{15} < \frac{n}{n+k} < \frac{7}{13} ?$$

(A) 108 (B) 109 (C) 110 (D) 111 (E) 112

12. Four consecutive even integers are removed from the sequence of integers $1, 2, \dots, n$, and the average of the remaining number is 51.5625.

Determine the largest integer removed.

(A) 28 (B) 30 (C) 32 (D) 34 (E) 36

13. Let ABCD be rhombus with $AB=5$. Suppose $AC \geq 6 \geq BD$, determine the maximum value of $AC+BD$.

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

14. Find the coefficient of x^{17} in the expansion of $(1+x^5+x^7)^{20}$.

(A) 3400 (B) 3410 (C) 3420 (D) 3430 (E) 3440

15. For how many real numbers a do the quadratic equations

$$x^2 + ax + 8a = 0$$

have only integral roots?

(A) 1 (B) 3 (C) 5 (D) 8 (E) 10

16. If x, y are non-zero numbers satisfying $x^2 + xy + y^2 = 0$, find the value of

$$\left(\frac{x}{x+y}\right)^{2007} + \left(\frac{y}{x+y}\right)^{2007}$$

- (A) 2 (B) 1 (C) 0 (D) -1 (E) -2

17. Let b a positive number. It is known that the equation $x^6 - 2bx^3 + b^2 - 100 = 0$ has exactly two real roots whose difference is 2.

Find the value of b .

- (A) $6\sqrt{3}$ (B) $5\sqrt{3}$ (C) $4\sqrt{3}$ (D) $6\sqrt{2}$ (E) $5\sqrt{2}$

18. In $\triangle ABC$, $\angle BAC = 40^\circ$ and $\angle ABC = 60^\circ$.

D and E are points on sides AC and AB respectively such that $\angle CBD = 40^\circ$ and $\angle BCE = 70^\circ$.

Let DB intersect CE at F and AF intersect BC at G.

Find $\angle GFC$.

- (A) 15° (B) 20° (C) 25° (D) 30° (E) 45°

19. Compute $1 + 3 + 5 + \dots + 2005 + 2007$

- (A) 1008160 (B) 1008106 (C) 1008016 (D) 1006018 (E) 1006081

20. Let a and b are positive integers $10 < a < 1001 < b < 2007$.

Then $\frac{a}{b}$ is largest if

- (A) $a=11,$
 $b=2006$ (B) $a=1000,$
 $b=2006$ (C) $a=11,$
 $b=1002$ (D) $a=1000,$
 $b=1002$ (E) $a=500,$
 $b=1504$

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