Name:_____ School:_____

1. Find the smallest natural number that is divisible by 2017 and whose decimal notation begins with 2016. **(12 Marks)**

Answer. 20161932.

Solution. Suppose this number n has $k + 4$ digits. Then $2016 \cdot 10^k \le n < 2017 \cdot 10^k$. Since n is divisible by 2017, it follows that $n \le 2017 \cdot 10^k - 2017$.

Hence $2017 \le (2017 - 2016)10^k = 10^k$, so $k \ge 4$. Thus the smallest such number is equal to $20170000 - 4 \cdot 2017$.

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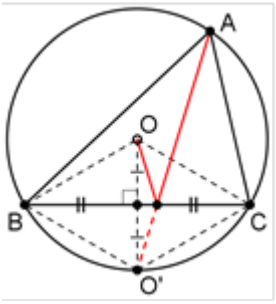
2 An acute-angled triangle ABC is such that $\angle A = 60°$. A billiard ball goes from vertex A along the bisector of angle A, reflects about the side BC according to the law "the angle of reflection equals the angle of incidence" and continues along a straight line without any further reflections. Prove that the path of the ball contains the circumcenter of triangle ABC. **(12 Marks)**

Solution. Reflect along the side BC the center O of the inscribed circle Ω of the triangle ABC. We get the point O'. Since $\angle BO'C = \angle BOC = 2\angle A = 120^{O} = 180^{O} - \angle A$, the point O' lies on Ω. Since O lies on the perpendicular bisector of BC, O' is the middle of the arc BC. It follows that the bisector of the angle A passes through O'. This means that after the reflection, the ball will pass through O.



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3. Is it possible that the sum and the product of 2017 integers are both equal to 2017?

(12 Marks)

Answer. Yes, it is possible.

Example . 2017, 1008 ones and 1008 minus ones.

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4. Compute $Sin^6\alpha + Cos^6\alpha$, if $Sin\alpha + Cos\alpha = m$? **(12 Marks)**

Solution: $(Sin\alpha + Cos\alpha)^2 = 1 + Sin2\alpha = m^2$ and $Sin2\alpha = m^2 - 1$.

$$Sin^6\alpha + Cos^6\alpha = (Sin^2\alpha)^3 + (Cos^2\alpha)^3 = Cos^4\alpha - Sin^2\alpha\ Cos^2\alpha + Sin^4\alpha =$$

$$1 - 3Sin^2\alpha\ Cos^2\alpha = 1 - \frac{3Sin^2 2\alpha}{4} = 1 - \frac{3(m^2-1)^2}{4}.$$

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5. For which values of n and m the number $10^{2n}+10^{n}+1$ is divisible by $10^{m}-1$?

(12 Marks)

Solution: For any n and m first number is not divisible by second since first number is divisible by 3 but second one is divisible by 9.