الجامعة الاسلامية العالمية ماليزيا

INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA

IIUM Mathematics Competition (IMC 2014)

LONG QUESTIONS

This Question Paper Consists of 11 Printed Pages with 10 Questions

Department of Computational and Theoretical Sciences
Kulliyyah of Science

The result is computed from the five problems with the highest scores.

Name:_____________________ School:___________________

1. Let ABC be a right angle triangle with sides a, $a + d$, and $a + 2d$ respectively.

Prove that d is equal to the radius of inscribed circle. **(6 Marks)**

Hint. A radius r of inscribed circle one can compute using the following formula $r = \frac{S}{p}$, where S is the area and p is a half –perimeter of given triangle.

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

2. Find all positive integers n such that the following ratio

$$\frac{20n+14}{14n+20}$$

take the integer values. **(6 Marks)**

Hint Select integer part of this ratio

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

3. Ahmad would like to buy totally 40 king's pictures of three type with cost 1 cent, 4 cent and 12 cent for one ringgit. How many king's pictures of each type he can buy? **(6 Marks)**

Hint Let x,y,z be the numbers of king's pictures with cost 1 cent, 4 cent and 12 cent respectively. Then $x+y+z=40$ and $1\cdot x+4\cdot y+12\cdot z=100$.

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

4. Solve the equation

$$x^2 - |x| = 12$$

where |x |is the absolute value *x*. **(6 Marks)**

Hint $x^2 = |x|^2$

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

5. Prove that

$$\sqrt{\underset{2n\ times}{111...1} - \underset{n\ times}{222...2}} = \underset{n\ times}{33...3}$$ **(6 Marks)**

Hint $\underset{2n\ times}{111...1} = 1 + 10 + 100 + ... + 10^{2n-1} = \frac{10^{2n}-1}{9}$

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

6. Let ABC be an isosceles triangle. Suppose that points K and L are chosen on its lateral sides AB and AC respectively so that $AK = CL$ and $\angle ALK + \angle LKB = 60^\circ$. Prove that $KL = BC$. **(12 Marks)**

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

7. Denote by $(a; b)$ the greatest common divisor of a and b. Let n be a positive integer such that $(n; n + 1) < (n; n + 2) < \dots < (n; n + 35)$.

Prove that $(n; n + 35) < (n; n + 36)$. **(12 Marks)**

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

8. Each of 100 given numbers was increased by 1. Then each number was increased by 1 once more. It is known that the first time the sum of the squares of the numbers was not changed. Find how this sum changed the second time.

(12 Marks)

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

9. Find all integer solutions (x,y) of the following equation

$$(x-2)(x-10) = 3^y$$

(12 Marks)

The result is computed from the five problems with the highest scores.

Name:______________________ School:____________________

10. A 7-digit passcode is called good if all digits are different. A safe box has a good passcode. The safe box opens if a good code is entered and one of its digits is equal to the corresponding digit of the passcode. Is there a method of opening the safe box with unknown passcode using less than 7 attempts? **(12 Marks)**