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1. A carpenter sells armchairs, book cases and cabinets. A person buys **8** armchairs, **11** bookcases and **2** cabinets and pays **RM875**. Another person buys **3** armchairs, **2** bookcases and **5** cabinets and pays **RM343**. Find the total cost of **1** armchair, **1** bookcase and **1** cabinet.

Solution:
Suppose 1 armchair cost **RM *x***, 1 bookcase cost **RM *y*** and 1 cabinet cost **RM *z***. THen

$$8x + 11y + 2z = 875 \qquad \text{----} \qquad (1)$$

$$3x + 2y + 5z = 343 \qquad \text{----} \qquad (2)$$

(1) + 3·(2) get

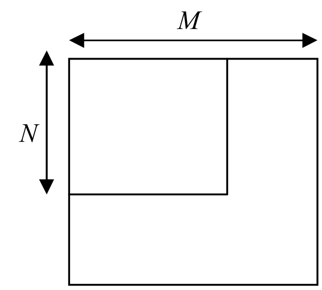
$$17x + 17y + 17z = 1904$$

So $x + y + z = 112$

Name:__________________________ School:__________________________

2. A square whose sides are of integral length is cut into 25 small squares whose sides are also of integral lengths. Exactly 24 of these smaller squares are unit squares. Find the area of the original square.

Solution:



Let M^2 is the area of the original square and N^2 is the smaller square in the figure.
Then

$$M^2 = N^2 + 24$$

$$M^2 - N^2 = 24$$

$$(M+N)(M-N) = 24$$

Both M, N has to be even or odd.

Consider all factors of 24: 1,2,3,4,6,8,12,24.
Only (2,12) and (4,6) pairs are both even.

If $M+N=6$ and $M-N=4$, then $M=5$ and $N=1$, which is not possible because if $N=1$, then there are total 25 unit squares, violating exactly 24 unit square.

If $M+N=12$ and $M-N=2$, then $M=7$ and $N=5$.
The are of the original square is 49 .

Name:__________________________ School:__________________________

3. Let 2007 even consecutive natural numbers be given.
   Prove that among any 42 numbers, chosen arbitrarily from these numbers, there exist two numbers, the positive difference of which is less than 100.

Solution:

Let $a_1 = 2k, a_2, a_3, ..., a_{2006}, a_{2007} = 2k + 4012$ be the sequence of the chosen natural numbers.

Then $a_{2007} = a_1 + 2 \cdot 2006 = a_1 + 4012$.

Let $b_1, b_2, b_3, ..., b_{41}, b_{42}$ be the sequence of the chosen natural numbers from $\{a_k\}$.

Then,

$$4012 = a_{2007} - a_1 > b_{42} - b_1$$
$$4012 > b_{42} - b_1$$

Also, we can write

$$\begin{aligned} b_{42} - b_1 &= b_{42} - b_{41} + b_{41} - b_{40} + b_{40} + ... - b_2 + b_2 - b_1 \\ &= (b_{42} - b_{41}) + (b_{41} - b_{40}) + ... + (b_2 - b_1) \end{aligned}$$

We prove this by contradiction. Suppose that there are no two distinct numbers b_i, b_j such that $|b_i - b_j| < 100$. Then $|b_i - b_j| \geq 100$ for any *i* and *j*.

$$\begin{aligned} b_{42} - b_1 &= (b_{42} - b_{41}) + (b_{41} - b_{40}) + ... + (b_2 - b_1) \\ &\geq 100 + 100 + ... + 100 \\ &= 4100 \\ b_{42} - b_1 &\geq 4100 \end{aligned}$$

We get a contradiction, since $4012 > b_{42} - b_1$.

Thus the supposition is false and there exist two numbers b_i and b_j satisfying the given property.

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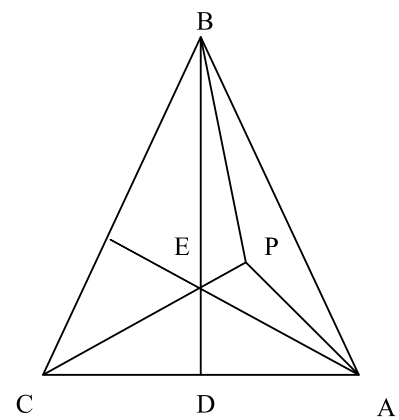
Name:__________________________ School:__________________________

4. In an isosceles triangle **ABC** (**AB**=**BC**), $\angle ABC = 80^\circ$.
   Let **P** be an interior point of the triangle, so that $\angle PAC = 40^\circ$ and $\angle ACP = 30^\circ$.
   Find $\angle BPC$.



Draw the height **BD**. Let **BD** intersect **CP** at E.
Here $\angle EAD = \angle ECD = 30^\circ \Rightarrow \angle APE = 110^\circ \Rightarrow \angle EAP = 10^\circ$.
Therefore $\angle DEA = \angle CED = \angle PEA = \angle BEP = 60^\circ$.
But $\angle ABE = 40^\circ$, $\angle DAB = 50^\circ$, $\angle PAB = 10^\circ$.
As a result in the triangle **ABE** the point **P** is the point is the point of intersection of bisectrices. Thus **B**P is also a bisectrix and
$\angle EBP = 20^\circ$.
So $\angle BPC = 180^\circ - \angle BEP - \angle EBP = 100^\circ$.

Name:___________________________ School:___________________________

5. Three nonzero real numbers are given. If they are written in any order as coefficient of a quadratic trinomial, then each of these trinomials has a real root. Is it true that in this case each of these trinomials has a positive root?
   One example of such trinomials is $ax^2 + bx + c$

Solution.

Let *a*, *b*, *c* be three nonzero numbers and let $|c|$ be the smallest absolute value.

Assume that one of these trinomials has no positive root.
There are not zero roots also, since all *a*, *b*, *c* are nonzero.
Since this trinomial has real roots, both of them should be negative and then all coefficients should be the same sign.
However $c^2 - 4ab \geq 0$, where real roots in any order, and $c^2 \leq |ab| < 4|ab|$ so that the numbers *a* and *b* have opposite sign.
This contradiction shows that all trinomials have a positive root.

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